

TERRAMETRA

LINEAR EQUATIONS

Terrametra Resources

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- Basic Terminology
- Linear Equations
- Identities, Conditional Equations, and Contradictions
- Solving for a Specified Variable (Literal Equations)



BASIC TERMINOLOGY

An *equation* is a statement that two expressions are *equal*.

$x + 2 = 9 \quad 11x = 5x + 6x \quad x^2 - 2x - 1 = 0$

To solve an equation means to find all numbers that make the equation a true statement. These numbers are the <u>solutions</u>, or <u>roots</u>, of the equation. A number that is a solution of an equation is said to <u>satisfy</u> the equation, and the solutions of an equation make up its <u>solution set</u>.

Equations with the same solution set are <u>equivalent equations</u>.



Let *a*, *b*, and *c* represent real numbers.

If
$$a = b$$
, then $a + c = b + c$.

The same number may be added to each side of an equation without changing the solution set.

Let *a*, *b*, and *c* represent real numbers.

If a = b and $c \neq 0$, then ac = bc.

Each side of an equation may be multiplied by the same nonzero number without changing the solution set.



LINEAR EQUATIONS (First Degree Equations)

A <u>linear equation</u> in one variable is an equation that can be written in the form ax + b = 0

Where *a* and *b* are real numbers with $a \neq 0$.

Linear equations Nonlinear equations $3x + \sqrt{2} = 0$ $\sqrt{x} + 2 = 5$

$\frac{3}{4}x = 12$	$\frac{1}{x} = -8$
0.5(x+3) = 2x - 6	$x^2 + 3x + 0.2 = 0$

A linear equation is also called a *first-degree equation* since the greatest degree of the variable is 1.

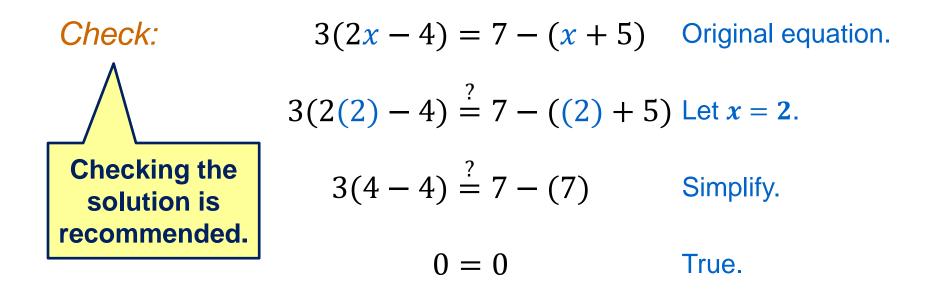


Example 1 Solving a Linear Equation

1(a)Solve:
$$3(2x-4) = 7 - (x+5)$$
Be careful
with signs.Solution: $6x - 12 = 7 - x - 5$ Distributive property. $6x - 12 = 2 - x$ Combine like terms. $6x - 12 + x = 2 - x + x$ Add x (both sides). $7x - 12 = 2$ Combine like terms. $7x - 12 + 12 = 2 + 12$ Add 12 (both sides). $7x = 14$ Combine like terms. $7x = 14$ Combine like terms. $\frac{7x}{7} = \frac{14}{7}$ Divide by 7 (both sides). $x = 2$



Example 1 Checking the Solution



The solution set is $\{2\}$.



Example 2 Solving a Linear Equation with Fractions

2(a) Solve:

$$\frac{2x+4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$$

Solution:

Multiply by **12** (the *LCD* of the fractions). Distribute the **12** to <u>*all*</u> terms within parentheses.

$$12\left(\frac{2x+4}{3} + \frac{1}{2}x\right) = 12\left(\frac{1}{4}x - \frac{7}{3}\right)$$

$$12\left(\frac{2x+4}{3}\right) + 12\left(\frac{1}{2}x\right) = 12\left(\frac{1}{4}x\right) - 12\left(\frac{7}{3}\right)$$

$$4(2x+4) + 6x = 3x - 28$$
Multiply.



Example 2 Solving a Linear Equation with Fractions

Solution (cont'd):

$$4(2x+4) + 6x = 3x - 28$$

$$8x + 16 + 6x = 3x - 28$$

11x + 16 = -28

14x + 16 = 3x - 28

Distributive property.

Combine like terms.

Subtract 3x (both sides).

Subtract 16 (both sides).

x = -4

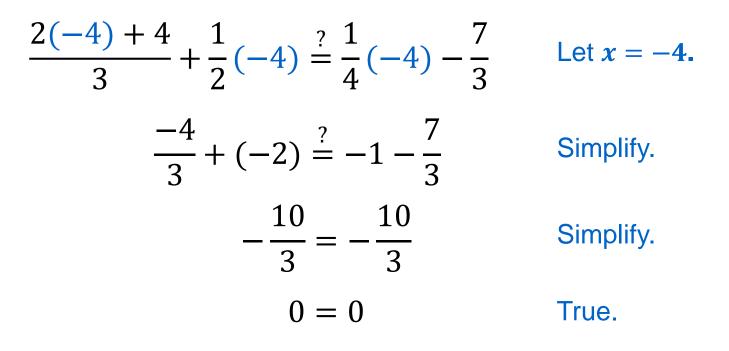
11x = -44

Divide by **11** (both sides).



Example 2 Checking the Solution

Check:



The solution set is $\{-4\}$.



IDENTITIES CONDITIONAL EQUATIONS CONTRADICTIONS

An equation satisfied by *all numbers* that are *meaningful replacements* for the variable is an *identity*.

3(x+1) = 3x+3

An equation that is satisfied by *some numbers*, but *not others*, is a *conditional equation*.

$$2x = 4$$

An equation that has no solution is a *contradiction*.

$$x = x + 1$$



3(a) Determine whether the equation is an *identity*, a *conditional equation*, or a *contradiction*.

$$-2(x+4) + 3x = x - 8$$

Solution: -2x - 8 + 3x = x - 8 Distributive property.

$$x - 8 = x - 8$$
 Combine like terms.

Subtract *x* and add 8. *(both sides)*

When a *true* statement such as 0 = 0 results, the equation is an *identity*, and the solution set is {all real numbers}.

0 = 0



3(b) Determine whether the equation is an *identity*, a *conditional equation*, or a *contradiction*.

5x - 4 = 11

Solution: 5x = 15 Add 4 (both sides).

x = 3 Divide by 5 (both sides).

This is a *conditional equation*, and its solution set is **{3**}.



3(c) Determine whether the equation is an *identity*, a *conditional equation*, or a *contradiction*.

$$3(3x-1) = 9x + 7$$

Solution: 9x - 3 = 9x + 7 D

Distributive property.

-3 = 7 Subtract 9x (both sides).

When a *false* statement such as -3 = 7 results, the equation is a *contradiction*, and the solution set is the **<u>empty set</u>** or **<u>null set</u>**, symbolized by $\{$ $\}$ or \emptyset .



IDENTIFYING TYPES of EQUATIONS

If solving a linear equation leads to a *true* statement such as 0 = 0, the equation is an <u>identity</u>. Its solution set is {all real numbers}.

If solving a linear equation leads to a *single* solution such as x = 3, the equation is <u>conditional</u>. Its solution set consists of a single element.

If solving a linear equation leads to a *false* statement such as -3 = 7, the equation is a <u>contradiction</u>. Its solution set is $\{$ $\}$ or \emptyset .



SIMPLE INTEREST FORMULA

A formula is an example of a *linear equation* (an equation involving letters). This is the formula for *simple interest*.

I is the $\longrightarrow I = P\gamma t \leftarrow$ variable for simple interest

P is the variable for dollars

r is the variable for annual interest rate

t is the

variable

for years



ACCUMULATED VALUE FORMULA

This formula gives the <u>future value</u>, or <u>maturity value</u>, A ("the accumulation") of P dollars ("the principle") invested for t years at an annual simple interest rate r.

$$\begin{array}{lll} A \text{ is the } \to A = P(1 + rt) \leftarrow t \text{ is the variable for } \\ \text{future or } \\ \text{maturity } \\ \text{value } \end{array} \begin{array}{l} P \text{ is the } \ & \downarrow \end{array} \begin{array}{l} \uparrow \\ \text{variable for } \\ \text{variable for } \end{array} \begin{array}{l} r \text{ is the variable for } \\ \text{variable for } \end{array} \end{array}$$

annual simple interest rate



Example 4 Solving for a Specified Variable

4(a) Solve for *t* :



Solution:

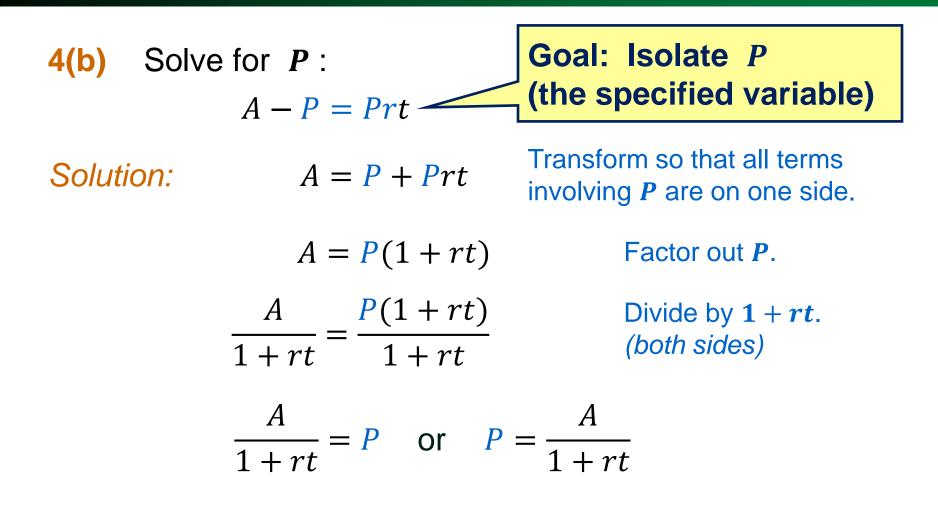
$$\frac{I}{Pr} = \frac{Prt}{Pr}$$

Divide by *Pr* (both sides).

$$\frac{I}{Pr} = t$$
 or $t = \frac{I}{Pr}$



Example 4 Solving for a Specified Variable





Example 4 Solving for a Specified Variable

